

Exercise 59

If $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} = 10$, find $\lim_{x \rightarrow 1} f(x)$.

Solution

Plugging in $x = 1$ or applying the Quotient Law is out of the question because the denominator becomes zero. In order for this limit to exist and be equal to 10, the numerator must contain a factor of $x - 1$ that cancels out with the one in the denominator. For simplicity, let

$$f(x) - 8 = A(x - 1),$$

where A is a constant to be determined.

$$\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} = 10$$

$$\lim_{x \rightarrow 1} \frac{A(x - 1)}{x - 1} = 10$$

$$\lim_{x \rightarrow 1} A = 10$$

$$A = 10$$

Substitute this value for A into the hypothesis and solve for $f(x)$.

$$f(x) - 8 = 10(x - 1)$$

$$f(x) - 8 = 10x - 10$$

$$f(x) = 10x - 2$$

Therefore,

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (10x - 2)$$

$$= 10(1) - 2$$

$$= 8.$$